Hana Krásná (née Spicakova) et al.: Solid Earth Tide Parameters from VLBI Measurements and FCN Analysis, IVS 2012 General Meeting Proceedings, p.405–409
http://ivscc.gsfc.nasa.gov/publications/gm2012/krasna.pdf

Solid Earth Tide Parameters from VLBI Measurements and FCN Analysis

Hana Krásná (née Spicakova), Johannes Böhm, Sigrid Böhm, Harald Schuh

Vienna University of Technology, Institute of Geodesy and Geophysics

Contact author: Hana Krásná (née Spicakova), e-mail: `hana.spicakova@tuwien.ac.at`

Abstract

In a common global adjustment of the 24-hour IVS sessions from 1984.0 till 2011.0 with the Vienna VLBI Software (VieVS), we have estimated simultaneously terrestrial reference frame (station positions and velocities), celestial reference frame (radio source positions), and Earth orientation parameters, together with complex Love and Shida numbers for diurnal tides and their frequency dependence caused by the resonance with the Free Core Nutation (FCN). As the FCN period is contained in the solid Earth tidal displacements and also in the motion of the Celestial Intermediate Pole w.r.t. the celestial reference system, it is determined from both phenomena as a common parameter in the global solution. Our estimated FCN period of -431.18 ± 0.10 sidereal days is slightly different from the value -431.39 sidereal days adopted in the IERS Conventions 2010.

1. Introduction

The site displacement caused by the solid Earth tides is characterized by Love and Shida numbers. In the Earth model with a fluid core and anelastic mantle, Love and Shida numbers of the diurnal tides are frequency dependent complex numbers (Wahr, 1981 [10]). The strong frequency dependence is produced by the Nearly Diurnal Free Wobble resonance (NDFW) associated with the Free Core Nutation (FCN). The resonant behavior of Love and Shida numbers in the diurnal tidal response of the solid Earth gives the possibility to estimate the FCN period directly from the displacement of the stations in the VLBI data analysis (Haas and Schuh, 1997 [4]). In the celestial reference frame the FCN is visible as a retrograde motion of the Earth's axis with a period of about 431 days and an amplitude of about one hundred microarcseconds. At present there are no models which could predict this motion in a rigorous way, so it is not included in the a priori precession-nutation model of the Earth axis. The motion is a major contributor to the VLBI residuals between the observed direction of the Celestial Intermediate Pole (CIP) in the celestial reference frame and the direction modeled by the precession-nutation model IAU 2006/2000A (Capitaine et al., 2003 [2] and Mathews et al., 2002 [7]) adopted in the current IERS Conventions 2010 (Petit and Luzum, 2010 [8]). In this work we estimate the FCN period from the solid Earth tidal (SET) displacement and also from the motion of the CIP as a common parameter in the global adjustment of VLBI measurements. The processing of 3360 24-hour IVS sessions from 1984.0 till 2011.0 was carried out with the VLBI analysis software VieVS (Böhm et al., 2012 [1]).

2. Love and Shida Numbers for the Diurnal Tides

We follow the representation of Love and Shida numbers employed by Mathews et al. (1995) [6], which is recommended by the IERS Conventions 2010. In the diurnal band, Love and Shida numbers are represented by a resonance formula (Equation (1)) as a function of the tidal excitation frequencies associated with the Chandler wobble (σ_{CW}), NDFW (σ_{NDFW}), and the Free Inner Core Nutation (σ_{FICN}):

$$L_f = L_0 + \frac{L_{CW}}{\sigma_f - \sigma_{CW}} + \frac{L_{NDFW}}{\sigma_f - \sigma_{NDFW}} + \frac{L_{FICN}}{\sigma_f - \sigma_{FICN}}. \tag{1}$$

L_f is a generic symbol for the frequency dependent Love (h) and Shida (l) numbers, and the numerical values for this equation are listed in Petit and Luzum (2010) [8]. The respective frequency dependent corrections to the displacement vector written in a complex form (Equation (2)) can be easily used for creating the partial derivatives with respect to the Love and Shida numbers or the FCN period:

$$\begin{aligned} \delta\vec{d_f} = &-\frac{3}{2}\sqrt{\frac{5}{24\pi}}H_f\left[\delta h_f^R \sin(\theta_f+\Lambda) + \delta h_f^I \cos(\theta_f+\Lambda)\right]\sin(2\Phi)\ \hat{r} \\ &-3\sqrt{\frac{5}{24\pi}}H_f\left[\delta l_f^R \cos(\theta_f+\Lambda) - \delta l_f^I \sin(\theta_f+\Lambda)\right]\sin(\Phi)\ \hat{e} \\ &-3\sqrt{\frac{5}{24\pi}}H_f\left[\delta l_f^R \sin(\theta_f+\Lambda) + \delta l_f^I \cos(\theta_f+\Lambda)\right]\cos(2\Phi)\ \hat{n} \end{aligned} \tag{2}$$

where:

H_f	is the Cartwright-Tayler amplitude (Cartwright and Tayler, 1971 [3]) of the tidal term with frequency f,
Φ, Λ	are geocentric latitude and longitude of the station,
θ_f	is the argument for the tidal constituent with frequency f, and
$\hat{r}, \hat{e}, \hat{n}$	represent the unit vectors in radial, east and north direction, respectively.

We estimated the frequency dependent Love and Shida numbers for twelve diurnal tides. Aside from the three strongest diurnal waves (K_1, O_1, and P_1) we included four tides (Q_1, M_1, π_1, K_1') with a lower frequency than that of the NDFW and five tides (ψ_1, ϕ_1, θ_1, J_1, Oo_1) with a higher frequency. The complex Love and Shida numbers were adjusted within a standard global solution of the VLBI data together with the terrestrial and celestial reference frames. Earth orientation parameters, zenith wet delays, horizontal tropospheric gradients, and clock parameters were reduced session-wise from the normal equations. The real and imaginary parts of the estimated Love numbers are shown in the third and fifth column of Table 1. Results for Shida numbers are listed in Table 2. The respective differences in the amplitudes of the vertical and horizontal displacement, when using our estimates of Love and Shida numbers instead of the values adopted in IERS Conventions 2010, are given in the fourth and sixth column. The expression follows from Equation (2):

$$\begin{pmatrix} \delta R_f^{ip} \\ \delta R_f^{op} \end{pmatrix} = -\frac{3}{2}\sqrt{\frac{5}{24\pi}}H_f \begin{pmatrix} \delta h_f^R \\ \delta h_f^I \end{pmatrix}, \tag{3}$$

$$\begin{pmatrix} \delta T_f^{ip} \\ \delta T_f^{op} \end{pmatrix} = -3\sqrt{\frac{5}{24\pi}}H_f \begin{pmatrix} \delta l_f^R \\ \delta l_f^I \end{pmatrix}. \tag{4}$$

The newly estimated Love and Shida numbers are very close to their theoretical values (see IERS Conventions 2010, Chapter 7.1.1). An exception is the tide θ_1, which shows a deviation of about 0.20 in the real part and about 0.15 in the imaginary part of the Love number. The estimated Shida number for this tide differs by about 0.05 in the real part from theory, whereas the imaginary part fits very well to the model. The inaccuracy at this tide is probably caused by the weak amplitude, which obstructs an accurate estimation. On the other hand it can be seen that the differences in $\delta R^{ip}_{\theta_1}$, $\delta R^{op}_{\theta_1}$, and $\delta T^{ip}_{\theta_1}$ are still small in the sub-millimeter range: -0.3 ± 0.1 mm, -0.2 ± 0.1 mm, and -0.2 ± 0.0 mm, respectively.

Table 1. Real and imaginary parts of the Love numbers for the diurnal tides computed by the software VieVS from VLBI data (1984.0 - 2011.0).

Name	Doodson number	$h_f^{(0)R}$	$\Delta\delta R_f^{ip}$ [mm]	$h_f^{(0)I}$	$\Delta\delta R_f^{op}$ [mm]
Q_1	135.655	0.6147 ± 0.0043	0.22 ± 0.08	-0.0087 ± 0.0043	-0.12 ± 0.08
O_1	145.555	0.6026 ± 0.0009	0.00 ± 0.09	-0.0013 ± 0.0008	0.11 ± 0.08
M_1	155.655	0.5888 ± 0.0101	0.09 ± 0.08	-0.0084 ± 0.0101	0.05 ± 0.08
π_1	162.556	0.5083 ± 0.0289	-0.22 ± 0.08	-0.0321 ± 0.0290	-0.08 ± 0.08
P_1	163.555	0.5816 ± 0.0017	-0.03 ± 0.08	0.0037 ± 0.0017	0.26 ± 0.08
K_1	165.555	0.5267 ± 0.0007	-0.08 ± 0.10	0.0041 ± 0.0007	-0.56 ± 0.10
K_1'	165.565	0.5294 ± 0.0043	-0.16 ± 0.08	0.0223 ± 0.0043	-0.42 ± 0.08
ψ_1	166.554	1.1224 ± 0.0701	-0.09 ± 0.08	0.3291 ± 0.0704	-0.36 ± 0.08
ϕ_1	167.555	0.7707 ± 0.0392	-0.22 ± 0.08	0.0007 ± 0.0392	-0.01 ± 0.08
θ_1	173.655	0.8093 ± 0.0515	-0.30 ± 0.08	0.1562 ± 0.0515	-0.24 ± 0.08
J_1	175.455	0.5988 ± 0.0098	0.09 ± 0.08	-0.0194 ± 0.0098	0.13 ± 0.08
Oo_1	185.555	0.6594 ± 0.0176	-0.23 ± 0.08	-0.0182 ± 0.0176	0.07 ± 0.08

3. Free Core Nutation

In this paper we estimate the FCN period at first only from the solid Earth tide displacement and then only from the motion of the CIP. Since we get from both solutions reasonable estimates of the FCN period, we connect both approaches and estimate the FCN period from both phenomena simultaneously. In Equation (5) we show the final partial derivative of the time delay τ w.r.t. the NDFW frequency σ_{NDFW} contained in the baseline $\boldsymbol{b}$ between two observing stations, and in the nutation matrix $\boldsymbol{dQ}$ which accounts for the effect of the FCN which is not included in the IAU 2006/2000A precession-nutation matrix $\boldsymbol{Q}_{(IAU)}$:

$$\begin{aligned}\frac{\partial\tau}{\partial\sigma_{NDFW}} &= \boldsymbol{k}(t)\cdot\frac{\partial\boldsymbol{dQ}(t)}{\partial\sigma_{NDFW}}\cdot\boldsymbol{Q}(t)_{(IAU)}\cdot\boldsymbol{R}(t)\cdot\boldsymbol{W}(t)\cdot\boldsymbol{b}(t)\\ &\quad+\boldsymbol{k}(t)\cdot\boldsymbol{dQ}(t)\cdot\boldsymbol{Q}(t)_{(IAU)}\cdot\boldsymbol{R}(t)\cdot\boldsymbol{W}(t)\cdot\frac{\partial\boldsymbol{b}(t)}{\partial\sigma_{NDFW}}.\end{aligned} \tag{5}$$

$\boldsymbol{k}$ is the source vector, and $\boldsymbol{R}$ and $\boldsymbol{W}$ are the remaining transformation matrices between the celestial and the terrestrial reference frame. The expression for the $\boldsymbol{dQ}$ as a simple rotation is shown in Equation (6)) (IERS Conventions 2010), and the X_{FCN} and Y_{FCN} coordinates were

Table 2. Real and imaginary parts of the Shida numbers for the diurnal tides computed by the software VieVS from VLBI data (1984.0 - 2011.0).

Name	Doodson number	$l_f^{(0)R}$	$\Delta\delta T_f^{ip}$ [mm]	$l_f^{(0)I}$	$\Delta\delta T_f^{op}$ [mm]
Q_1	135.655	0.0870 ± 0.0010	0.09 ± 0.04	-0.0027 ± 0.0010	-0.08 ± 0.04
O_1	145.555	0.0858 ± 0.0002	0.20 ± 0.04	-0.0006 ± 0.0002	0.02 ± 0.04
M_1	155.655	0.0815 ± 0.0025	0.05 ± 0.04	-0.0040 ± 0.0025	0.05 ± 0.04
π_1	162.556	0.0827 ± 0.0072	-0.01 ± 0.04	-0.0028 ± 0.0072	-0.01 ± 0.04
P_1	163.555	0.0864 ± 0.0004	0.08 ± 0.04	-0.0009 ± 0.0004	-0.02 ± 0.04
K_1	165.555	0.0881 ± 0.0003	-0.27 ± 0.08	-0.0008 ± 0.0003	0.02 ± 0.08
K_1'	165.565	0.0912 ± 0.0011	-0.15 ± 0.04	0.0027 ± 0.0011	-0.13 ± 0.04
ψ_1	166.554	0.0832 ± 0.0175	-0.03 ± 0.04	0.0409 ± 0.0175	-0.10 ± 0.04
ϕ_1	167.555	0.1052 ± 0.0098	-0.09 ± 0.04	-0.0273 ± 0.0098	0.11 ± 0.04
θ_1	173.655	0.1352 ± 0.0129	-0.15 ± 0.04	0.0026 ± 0.0129	-0.01 ± 0.04
J_1	175.455	0.0833 ± 0.0025	0.02 ± 0.04	0.0043 ± 0.0025	-0.08 ± 0.04
Oo_1	185.555	0.0856 ± 0.0045	-0.01 ± 0.04	-0.0050 ± 0.0044	0.04 ± 0.04

taken from the empirical FCN model of Lambert (2007) [5] giving the variable sine and cosine amplitudes of the harmonic oscillation in a one year resolution:

$$\boldsymbol{dQ} = \begin{bmatrix} 1 & 0 & X_{FCN} \\ 0 & 1 & Y_{FCN} \\ -X_{FCN} & -Y_{FCN} & 1 \end{bmatrix}. \tag{6}$$

In Table 3 estimates of the FCN period from all solutions are listed. In solution 1 the FCN period is estimated only from the solid Earth tidal displacement. The partial derivative follows from Equations (1) and (2). The FCN period in solutions 2 and 3 is derived only from the motion of the CIP, whereas in solution 3 the averaged cosine and sine amplitude (A_C, A_S) of the FCN are estimated as additional global parameters. The final solution 4 provides the FCN period derived simultaneously from both phenomena.

Table 3. Period (P), amplitude (A), and phase (Φ) of FCN estimated within global solutions by the software VieVS.

Solution	P [sid. days]	A_C [μas]	A_S [μas]	A [μas]	Φ [deg]
1 (solid Earth tides (SET))	-431.23 ± 2.44	-	-	-	-
2 (CIP)	-431.12 ± 0.06	-	-	-	-
3 (CIP — est. ampl.)	-431.17 ± 0.09	65 ± 1	35 ± 1	73 ± 1	28.03 ± 1.62
4 (SET + CIP — est. ampl.)	-431.18 ± 0.10	64 ± 1	34 ± 1	73 ± 1	27.90 ± 1.62

4. Conclusions

In this paper we focused on the estimation of Love and Shida numbers for twelve diurnal tides and on the estimation of the FCN period. The respective differences in the individual displacement amplitudes based on the theoretical and estimated Love and Shida numbers do not exceed 0.3 mm. The total difference to the theoretical displacement summed over the absolute values of all twelve diurnal waves reaches 1.73 mm in the vertical direction and 1.15 mm in the horizontal direction. The final value for the FCN period is derived simultaneously from the solid Earth tidal displacement and from the motion of the CIP. Its estimated value of -431.18 ± 0.10 sidereal days differs slightly from the conventional value -431.39 sidereal days given in Petit and Luzum (2010) [8].

Acknowledgements

The authors want to acknowledge the International VLBI Service for Geodesy and Astrometry (IVS) (Schlüter and Behrend, 2007 [9]) and all its components for providing VLBI data. Hana Krásná and Sigrid Böhm work within FWF-Project P23143 (Integrated VLBI).

References

[1] Böhm, J., S. Böhm, T. Nilsson, A. Pany, L. Plank, H. Spicakova, K. Teke, and H. Schuh (2012). The new Vienna VLBI Software VieVS. IAG Symposium 2009, Vol. 136. S. Kenyon, M.C. Pacino and U. Marti (ed.). pp. 1007-1012. ISBN 978-3-642-20337-4.

[2] Capitaine, N., P.T. Wallace, and J. Chapront (2003). Expressions for IAU 2000 precession quantities. Astron. Astrophys., Vol. 412/2. pp. 567-586. doi: 10.1051/0004-6361:20031539.

[3] Cartwright, D.E., and R.J. Tayler (1971). New computations of the tide-generating potential. Geophys. J. Roy. astr. Soc., Vol. 23/1. pp. 45-74. doi: 10.1111/j.1365-246X.

[4] Haas, R., and H. Schuh (1997). Determination of Tidal Parameters from VLBI. Marees Terrestres Bulletin D'Informations, Vol. 127. pp. 9778-9786.

[5] Lambert, S.B. (2007). Empirical modeling of the retrograde Free Core Nutation. Technical Note, ftp://hpiers.obspm.fr/eop-pc/models/fcn/notice.pdf

[6] Mathews, P.M., B.A. Buffett, and I.I. Shapiro (1995). Love numbers for a rotating spheroidal Earth: New definitions and numerical values. Geophys. Res. Lett., Vol. 22/5. pp. 579-582. doi: 10.1029/95GL00161.

[7] Mathews, P.M., T.A. Herring, and B.A. Buffet (2002). Modeling of nutation and precession: New nutation series for nonrigid Earth, and insights into the Earth's Interior. J. Geophys. Res., Vol. 107/B4, 2068. 26 pp. doi: 10.1029/2001JB000390.

[8] Petit, G., and B. Luzum (2010). IERS Conventions (2010). International Earth Rotation and Reference Systems Service (IERS). IERS Technical Note, No. 36, Frankfurt am Main, Germany: Verlag des Bundesamtes für Kartographie und Geodäsie, ISBN 1019-4568.

[9] Schlüter, W., and D. Behrend (2007). The International VLBI Service for Geodesy and Astrometry (IVS): current capabilities and future prospects. Journal of Geodesy, Vol. 81/6-8. pp. 379-387. doi: 10.1007/s00190-006-0131-z.

[10] Wahr, J. M. (1981). Body tides on an elliptical rotating, elastic and oceanless Earth, Geophys. J. R. Astron. Soc., 64, pp. 677-703.